Math 113 homework due 2/27

If you would know strength and patience, welcome the company of trees.
– Hal Borland
(of course he was talking about our kind of trees!)

(1) Read the rest of chapter 14 in the course notes

(2) Prove that the *chromatic number* of K_n is n. (this is the minimum number of colors required to color each vertex so that no two adjacent vertices are the same color)

(3) If G has chromatic number n, is it *necessary* that G contains a K_n subgraph? Hint: find a graph with chromatic number 3 that has no triangles. Hint #2: look at the next problem for inspiration

(4) A *bipartite graph* is a graph with chromatic number 2. Prove that every cycle (circuit) in a bipartite graph contains an even number of vertices. (Hint: draw some examples first and observe the circuits carefully).

(5) 14.12 from the course notes. (you can ignore the part about question 14.9, unless you want to discuss it in tutorial)

(6) How many non-isomorphic trees with 6 vertices are there? Draw them.

(7) Prove that connecting any two separate trees with a single edge makes a new tree. You may use the theorem we proved in class.

(8) Are all trees planar graphs? Why?

(9) We saw that in a tree, there is exactly one path from each vertex to each other. Suppose you have a graph G where there are *exactly two* paths from each vertex to each other. What must G look like? (hint: again, draw some examples with a small number of vertices).

(10) (to be done in tutorial) Suppose G is a graph with n vertices. Prove that the following are equivalent statements:
  (a) The graph is K_n
  (b) $\overline{G}$ has no edges
  (c) The total number of edges in G is $\frac{(n)(n-1)}{2}$
  (d) Every vertex has degree $n-1$